Maintaining Continuous Low Orbit Flight by Using In-Situ Atmospheric Gases for Propellant

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Abstract. An analysis is performed to determine the requirements for spacecraft propulsion systems operating in free-molecule flow and employing ambient atmospheric neutral gas as a propellant. It is concluded that some form of electric propulsion is required and the requirements for an electrostatic thruster system to be applied for this purpose are also discussed. An example thruster system based on an orbitron electrostatic electron trap is evaluated in several modes of operation. The selected configurations are shown to be unable to provide in any obvious way the required ionization fractions. The technological and physical limits that require solutions for this type of device are discussed.

INTRODUCTION

a.) Background

Prolonged flight in very low planetary orbits (less than 300 km for Earth or less than 200 km for Mars) using atmospheric species to supply the necessary propellant for drag make-up, as well as orbit raising or lowering and plane changes presents interesting opportunities. An extended very low altitude orbit and maneuvering capability permits a loitering orbital platform to have timely, high resolution access to all planetary surface locations. The dynamic characteristics of mid-altitude planetary atmospheres (on Earth from 100 to 300 km) are difficult to monitor, seriously limiting our understanding of this important region. Continuous very low altitude orbital flight would be a major step in the study of planetary atmospheres.

A specific impulse, I_{sp} , of about 800s is required for a thruster that uses collected ambient species from Earth's atmosphere as propellant, simply to make up for the drag associated with the collection. To negate the drag of an entire space vehicle (spacecraft and thruster), an I_{sp} significantly higher than 800s is required. Assume a loss-free system and a situation where the frontal area of the spacecraft is equal to but separate from that of the propellant collection area, an I_{sp} of 1600s would be required. An alternative possibility is to act on the ambient gas in transit, thus avoiding the drag overhead associated with collection. For this strategy applied to the previous example, achieving thrust equal to the drag of the entire space vehicle would still require an I_{sp} of 800s (for convenience the I_{sp} is based on the mass flow of ambient neutrals incident on the thruster). The high I_{sp} requirements (even for loss-free systems) indicate that some form of electric propulsion is mandatory.

b.) Previous Work and Results

The possibility of applying ambient neutral gas as propellant for spacecraft propulsion is an attractive concept, it has been investigated by others, primarily in the 50s and 60s¹⁻⁴. The associated concepts have been roughly

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categorized into two groups by Laux⁴, recombination ramjets and fluid accumulators. Recombination ramjets use the recombination of atomic oxygen collected from the atmosphere above 90 km altitude to produce the power required for propulsion. Recombination ramjets were shown to have possible operational Mach numbers less than 4. Fluid accumulators collect and cryogenically store components of the atmosphere for use by the satellite's propulsion systems and possibly by other satellites. Fluid accumulators were designed to operate at orbital velocities. It was concluded that the thruster system must provide specific impulses from 1000s to 1800s. The major problem requiring a solution for these types of devices was shown to be the efficient collection of the atmospheric gases at orbital velocities. Fly-through operation, without the need for large diffusers for collection, would avoid this problem (although other problems may be encountered).

INITIAL SYSTEMS STUDY RESULTS FOR A THERMOSPHERIC CRUISER

a.) Some General Systems Considerations

If a Thermospheric Cruiser, an entire space vehicle employing a thruster system using ambient neutral gas for propellant, operates with a thrust-to-drag ratio for the entire space vehicle, T/D_{sv} , of less than one and is operated continuously, degradation of the space vehicle's orbit will be decreased. With the system operating continuously at a T/D_{sv} of exactly one, the initial orbit is maintained with only small perturbations. If the thruster system is capable of T/D_{sv} greater than one, there are several additional operational choices. The thruster system could be run in a pulse mode, possibly sharing power sources with other space vehicle operations, to maintain its orbit. It could also be run continuously using the extra thrust to; provide orbital maneuvering (altitude and plane change) or attitude control capabilities, and/or permit accumulation of propellant. The following is a list of some of the main features of a Thermospheric Cruiser operating at different thrust to drag ratios.

<u>Primary Features of a Thermospheric Cruiser System (T/D_{sv} $\cong 1$):</u>

- Infinite or increased low altitude space vehicle lifetimes
 Braking thrust operation for lowering orbit
- No on-board propellant mass or volume required
- Possibility of a simple, low mass system
- Possibility of operating in atmospheres of other planets

Optional $(T/D_{sv} >> 1)$ Features:

- Share existing power supplies, average $T/D_{sv} \cong 1$ Electrostatic thrust vectoring
- Accumulate propellant that could be used for higher thrust maneuvers

b.) Generic Calculations to Define Performance of Thermospheric Cruisers

A Thermospheric Cruiser will be exposed to an atmosphere with an average number density, n, and molecular mass, m, at the altitude of the space vehicle. For an initial estimate the hypersonic approximation is adopted and velocity of the molecules relative to the spacecraft is the orbital velocity. If the thruster system is aligned with the flow (Fig. 3a) a certain fraction of the ambient molecules will enter the open area of the thruster system and the rest will interact with the space vehicle. A certain fraction of the ambient molecules, X, will be used for propulsive purposes and a certain fraction, X_d , will impart a drag force on the space vehicle. These fractions can be based on the thruster system or the entire space vehicle as long as they are consistent with the definition of the mass flow. Particles that don't contribute to either of these mechanisms have no net effect on the device. The shear drag on the sides of the space vehicle is neglected and fully accommodated diffuse reflections are considered on the ram surface. The thrust-to-drag ratio as a function of environmental and thruster system parameters is given by

$$T/D = \left(\sqrt{m} \cdot u_{orbit}\right)^{-1} \cdot \left(\sqrt{2qV_{acc}}X_t/X_d\right) \tag{1}$$

where u_{orbit} is the orbital velocity at that altitude and V_{acc} is the accelerating voltage applied by the thruster system. The specific impulse, I_{sp} , produced by the thruster system based on the mass flow of the ambient gas incident on the cross-sectional area of the thruster system is given by

$$I_{sp} = \frac{T}{\dot{M}_{thruster} g_o} = \frac{X_{t,thruster}}{g_o \sqrt{m}} \cdot \sqrt{2q V_{acc}}$$
 (2)

Figure 1 shows the possible thrust-to-drag ratios for Thermospheric Cruisers with I_{sp} 's of 1000s and 3000s. Fractions of incoming stopped less than 0.1 represent the thruster system while fractions between 0.5 and 0.75 represent an entire space vehicle. These results indicate that the required active area of the thruster system is likely to be similar to the ram area of the rest of the spacecraft. Specific impulses of the order of 3000s are required for these ionization levels. The fraction of incoming neutrals used for thrust purposes must exceed 10% to be feasible.

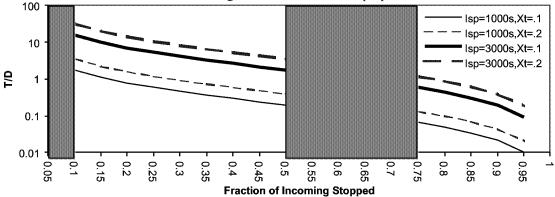


FIGURE 1. Possible T/D for a Thermospheric Cruiser, X, is the fraction of incoming molecules used for propulsion

If a thruster system provides a total space vehicle thrust to drag ratio greater than 1 the extra thrust can be used to counteract drag associated with collecting a fraction of the incoming ambient gas for later use as propellant. If the collection is 100% efficient the collection rate expressed as the equivalent velocity increment per unit time, shown in Fig. 2 for drag coefficient C_d =2.5 and ballistic coefficient B=65 kg/m², can be written in terms of the thruster system performance without collection as

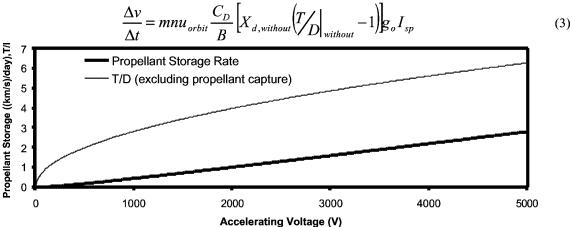


FIGURE 2. Possible Propellant Collection Rates

c.) Operational Envelope

The operational altitude envelope for a Thermospheric Cruiser based on an electrostatic propulsion system can be estimated by considering the factors that will limit the ambient neutral density in which the device can operate. The altitude at which the device is no longer required for drag makeup defines the upper altitude limit. The lower altitude limit may be due to heating effects on ram surfaces and solar panel drag. The estimates for the operational altitude envelope are summarized in Table 1.

Consideration	Assumptions	Earth Altitude	Mars Altitude
Ballistic Satellite Lifetime	Based on deorbit due to drag	~600 km	~550 km
	10 yr, solar max, ball coef. 65 kg/m ²		
Drag Heating Effects	Flat plate normal to incoming flow	~120 km	~100 km
Solar Panel Drag vs.	Bounds placed by flat plates	~300 km normal	~200 km normal
Power Produced	normal and parallel to flow	~65 km parallel	~50 km parallel

TABLE 1. Estimated Operational Altitudes for a Planetary Orbital Cruiser

The results indicate that the operational altitude envelope can be sensitive to the mission and the design of the entire space vehicle. They also indicate that there is a possible altitude envelope of 120 to 600 km on Earth and 100 to 550 km on Mars for a Thermospheric Cruiser, assuming the details of a specific orbit and spacecraft designs permit sufficient power to be available.

ANALYSIS OF AN EXAMPLE THERMOSPERIC CRUISER

a.) Orbitron Electron Containment

As illustrated in Fig. 1 electric propulsion is the only viable option for a thruster system using atmospheric gas as a propellant. To go further in the study a specific form of electric propulsion must be chosen. Electrostatic systems avoid the added mass of a magnetic system and the complexity associated with other types of electric propulsion systems. A typical electrostatic system would use electron impact ionization to create ions and then accelerate them through accelerating grids. If such a system was operated in the ram mode, for reasons previously discussed, then the electron number density must be maximized to increase the chance of ionizing a neutral molecule as it quickly travels through the device at the orbital velocity. This indicates that some sort of electron confinement scheme is likely required to provide the high electron number densities. One possible electrostatic electron trap is the orbitron. Their characteristics are well understood and they have been applied in several important devices such as ion pressure gages and electrostatic ion pumps⁵⁻⁸.

Orbitrons confine orbiting electrons between two concentric cylindrical electrodes. Electrons are injected into the orbitron with sufficient angular momentum to orbit the anode, but insufficient energy to escape the cathode mesh. They should be injected with a spatial distribution that maximizes the average electron density and minimizes issues such as space charge. The electrons continue to orbit, reflecting at each end due to the electron containment grids (further discussed in section b), until they escape or strike the anode due to a collisional transfer of energy or small perturbations in the electric field.

There is a logarithmic variation of potential between the concentric cylindrical electrodes, producing the condition that all circular orbits in the orbitron have the same kinetic energy⁸. Thus a large fraction of the trapped electrons can be near the maximum ionization cross-section for the atmospheric gas propellant. The circular orbital velocity for trapped electrons in an orbitron is given by

$$v_c = \sqrt{\frac{qV}{m_e \cdot \ln(r_2/r_1)}} \tag{4}$$

where m_e is the electron mass, V is the potential difference between the cathode and the anode, r_2 is the radius or the outer electrode, cathode mesh, and r_1 is the radius of the inner electrode, anode. The efficiency of the electron trap for ionization is conveniently measured by the maximum electron number density that it can confine. The total bound positive charge on the anode places an upper limit on the total allowable electron charge orbiting the anode and is shown by

$$Q = \frac{2\pi\varepsilon_o Vl}{\ln\left(r_2/r_1\right)} \tag{5}$$

where ε_0 is the permittivity, and l is the length of the device. The corresponding electron number density is given by

$$n_e = \frac{2\varepsilon_o V}{q \cdot \ln(r_2 / r_1)(r_2^2 - r_1^2)}$$
 (6)

b.) Description of Complete Thruster

A cross-section of the Molecular Electrostatic Ram Thruster, based on a modified version of a triode orbitron, is shown in Fig. 3a. Several additional grids are added to the orbitron to operate it as a complete thruster. All of the grids are highly transparent to minimize neutral drag and ion losses. Once an ion is created in the orbitron ionization chamber it is strongly accelerated radially towards the highly transparent cathode mesh and escapes the ionization chamber. A third concentric mesh cylinder, an ion containment grid, is added to the orbitron ionization chamber and

is held at a positive voltage relative to the cathode mesh (similar in magnitude to the cathode-anode potential) to decrease the radial velocity of the ions, repelling them back into the ionization chamber. If the device is operated in the ram mode the ions will have an initial longitudinal velocity relative to the thruster system of the orbital speed of the spacecraft. The ions will convect through the device as they cycle through the cathode mesh and leave the device after several cycles. Accelerating grids at the downstream end of the thruster provide the required specific impulse.

The Molecular Electrostatic Ram Thruster electrostatically shields the surrounding environment from the applied potentials inside the thruster system. The shielding electrodes and outer accelerating grid are set at the local environment's plasma potential. The thruster's radial potential variation is shown in Fig. 3b. The acceleration of the internally generated ions to provide thrust is between the potential at the ion's radial position and the plasma potential at the downstream side of the accelerating grids. This produces a radially varying exhaust velocity.

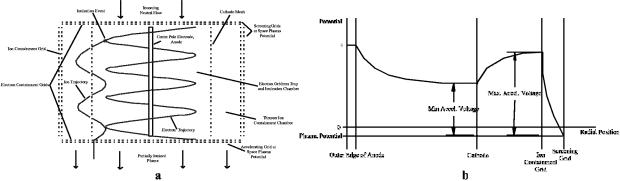


FIGURE 3a,b. Radial Potential Curve of Proposed Molecular Fly-Through Electrostatic Ram Thruster

Any ambient ions that enter the device with sufficient energy to overcome the discharge chamber potential are first decelerated to the discharge chamber potential and then reaccelerated by the accelerating grids to their original velocity when they are again at space plasma potential. Ambient ions with insufficient energy are reflected by the ram screening grid and contribute, typically a negligible amount, to the net drag acting on the device. Ambient ions can not be used to provide net thrust in an electrostatic system.

The ionization degree produced by the Molecular Electrostatic Ram Thruster before losses is given by the ratio of the ion production rate, I, to the rate at which neutrals flow through the device, \aleph ,

$$X_{i} = \frac{I}{\aleph} = \frac{\sigma Q v_{c}}{u_{orbit} \cdot A_{front} \cdot q}$$
(7)

Where σ is the electron impact ionization cross-section, Q is the total charge of the electrons orbiting the anode in the device, and A_{front} is the cross-sectional open area of the ionization chamber. The maximum ionization fraction for a given operating condition can then be expressed in terms of the physical dimensions and applied voltages as

$$X_{i} = \frac{\sigma 2\varepsilon_{o} V^{3/2} \tau}{r_{2}^{2} \sqrt{qm} \cdot \ln\left[\left(r_{2} / r_{1}\right)\right]^{3/2}}$$
(8)

where τ is the average residence time for a neutral molecule in the device. The ratio of the degree of ionization of the atmospheric gas passing through the orbitron can be scaled for a realization, α , relative to a reference orbitron

$$\frac{(X_i)_{\alpha}}{(X_i)_r} = \left(\frac{V_{\alpha}}{V_r}\right)^{3/2} \cdot \left(\frac{\ln(r_2/r_1)_{\alpha}}{\ln(r_2/r_1)_r}\right)^{3/2} \cdot \frac{(r_2^2 - r_1^2)_r}{(r_2^2 - r_1^2)_{\alpha}} \cdot \frac{\sigma_{\alpha}}{\sigma_r} \cdot \frac{\tau_{\alpha}}{\tau_r}$$
(9)

From this analysis it is clear that the primary factors in determining the maximum ionization fraction produced by the described device is the applied potential, the average neutral residence time, and the radii.

Once an ion is created it must be efficiently transported to the location of the accelerating grids. Along the way ions can be lost by striking a physical surface; an electrode, ion containment grid or the accelerating grids. The primary loss mechanism will likely be ion collisions with the cathode mesh due to the number of times that an average ion passes through the mesh as it travels along the device. The qualitative effects on the ion losses can be studied by viewing the ratio of the longitudinal transit time to the time required to travel from the anode to the cathode mesh. This relation is given by

$$X_{l,cathode} \sim \frac{l}{u_{orbit}} \cdot \left(\frac{\pi m r_1^2 \ln(r_2/r_1)^3}{2qV}\right)^{-1/2} \cdot erf(1)$$
(10)

c.) Performance Estimates

The maximum allowable electron number density in the orbitron ionization region has been earlier identified as one of the most important parameters in determining the ionization fraction provided by the orbitron and thus the overall performance of the Molecular Fly-Through Electrostatic Ram Thruster. The initial focus of the study was therefor to estimate the maximum allowable electron number density in the orbitron ionization region and to determine the qualitative effect of the electron number density on the electrostatic potential and trajectories of the ions and orbiting electrons near this limit. Initially a single device with dimensions on order of typical spacecraft dimensions was chosen as the simplest possible design. The anode radius was 1cm, the cathode radius was 17cm, the outer radius of the device was 25cm and it was 1.0m long. The applied potential on the orbitron, 600V, was chosen to provide circular orbits for electrons with kinetic energy of 100 eV. Using equation 8 the maximum electron number density is 7.2E11 m³. For flight speeds of 7.4 km/s the flight time is $\tau = 1.36E-4$ s. For these values the ionization fraction using atomic oxygen ($\sigma = 1E-20$ m²) is $(X_i)_r = 6.2E-6$. The ionization produced by this configuration of the Molecular Electrostatic Ram Thruster is far to low for application. Fig. 4 shows the variation of the electrostatic potential with the electron number density in the device as a fraction of the analytically determined maximum electron number density.

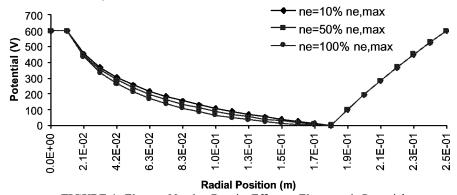


FIGURE 4. Electron Number Density Effect on Electrostatic Potential

Deviations from the applied potential become noticeable as the electron number densities approach the analytically determined limit. At these number densities the electric field is stronger than the low electron number density cases near the anode, but weaker near the cathode mesh. Fig. 5a shows electron trajectories with initial velocities that would provide a circular orbit if no space charge was present for different electron number densities inside the orbitron ionization chamber. Fig. 5b shows electron trajectories that would have elliptical orbits if no space charge was present.

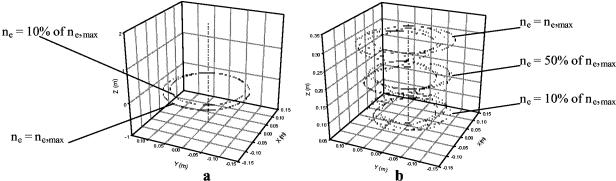


FIGURE 5. Electron Number Density Effect on Electron Trajectory

At an electron number density 10% of the maximum the space charge has a negligible effect on the trajectory of the orbiting electrons and the trajectory in Fig. 5a remains circular. As the electron number density is increased the

space charge screens more of the applied potential at any given radial position and the same initial condition produces elliptical orbits. For orbits with higher radii the screening of the applied potential will begin to cause some of the 100 eV electrons to be lost through the cathode mesh. Fig. 6 shows the electron number density effect on ion trajectories.

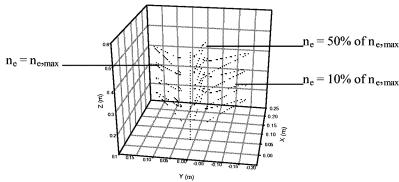


FIGURE 6. Electron Number Density Effect on Ion Trajectory

The space charge has only a minor effect on the ion trajectories. As the electron number density is increased the electric field in the orbitron ionization region near the cathode mesh is decreased allowing the ion to have a slightly higher period for its travel in the radial direction which slightly decreases the number of oscillations through the cathode mesh. The characteristics of the system will be similar for other configurations as the electron number density approaches the analytical limit for each specific case.

In order to investigate raising the ionization fraction consider the rather extreme case of making $(r_2)_{\alpha}=0.01$ m, $\tau_{\alpha}=\tau_r$, $V_{\alpha}=5000$ V, $\sigma_{\alpha}=0.7$ σ_r (due to the increased V). From Eq. 9 $(X_i)_{\alpha}/(X_i)_r=5.2E3$. This produces a dramatic increase, giving $(X_i)_{\alpha}=0.032$. It is still too low to be feasible, but indicates the magnitude of the effects due to scaling. While the ionization fraction increased to 0.03 and is approaching the target of $X_i>0.1$, it does not leave much room for unanticipated losses. The most critical issue associated with a small diameter, long orbitron is large numbers of ion crossings of the cathode. A modification of the basic orbitron thruster configuration shown in Fig. 3a may be required to provide an axial acceleration (using a potential gradient on the containment grid) of the ions once they are in the cathode-containment grid space. Another issue would be that in order to obtain sufficient total thrust for a large frontal area satellite would require a large number of the small orbitron thrusters.

Another possibility for increasing the ionization produced by the device would be to operate it in the stagnation region in front of the ram surfaces of the spacecraft. The particles reflecting off the spacecraft would have average velocities an order a magnitude less than those in the free stream thus increasing the transit time of the particles and consequently the ionization fraction. The reflected particles would have imparted their momentum to the spacecraft regardless of whether the thruster system was present so there is no drag cost of the thruster system. There are several difficulties in implementing such a configuration, however. In the ram mode the initial velocity of the ions carry them through the device allowing the ions to be efficiently transported to the accelerating grids. For the stagnation region configuration fields must be applied to transport the ions to the accelerating grid region. This is a complicated matter since the fields would also affect the electron confinement.

Properly designed end containment electrodes are required for both configurations to provide the most efficient containment of the electrons and also to shield the ionization chamber from the plasma potential applied on the outer accelerating grid. An initial configuration biases the containment electrodes 10 volts negative to the potential that would exist at the radial position if it were infinitely long. Fig. 7a shows the trajectory for a 100 eV electron injected at a radial position of 0.05m and a longitudinal position half-way down the device. The electron number density is 5E12 m³. The longitudinal velocity is 10% of the orbital velocity. For these conditions the electron is contained within the longitudinal positions of 0.145m and 0.355m. Fig 7b shows the same trajectories for an electron with a longitudinal velocity that is 50% of the circular velocity. At these energies the electron easily escapes the end of the device.

The results indicate that, by biasing the containment grids at a potential slightly negative to that which would occur in an infinitely long orbitron ionization chamber, the electrons can be trapped. Applying too strong of a potential on the containment grids would confine the electrons to only a small region in the center of the device. There is a design optimum since for such a configuration, but further investigation is not warranted until a configuration can be found that is feasible. More complicated schemes would be required for the stagnation region case since they must also move the ions to the accelerating grid region.

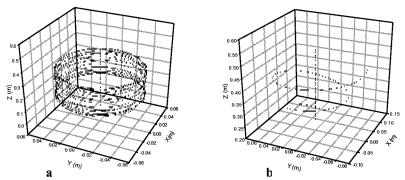


FIGURE 7. Electron Trajectories With End Effects

d.) Computational Method

The computational study of the Molecular Fly-Through Electrostatic Ram Thruster in both configurations was accomplished using an axially symmetric Particle-in-Cell (PIC) simulation code. The electric fields are defined on the spatial grid given by Birdsall and Langdon⁹. The potential and the charge density are defined on the full grid points while the electric fields are defined at the half grid points. Each spatial gridpoint can be defined as solid or open allowing for the definition of physical objects. The code also allows point-by-point definition of either Dirichlet or Neumann boundary conditions on the domain boundary.

Initial investigations were used to study the general characteristics of the Molecular Fly-Through Electrostatic Ram Thruster in both configurations. A constant electron number density was assumed inside the orbitron ionization chamber to represent the optimal operation of the device. This assumption also removes the effects of the specific electron injection method on the characteristics. The electron number density was assumed to be negligibly small in the ion containment region and outside the device. It is estimated that the ion number density is more than 3 orders of magnitude smaller than the electron number density and therefor can be neglected when calculating the electrostatic characteristics inside the ionization chamber. The ion number density also contributes a negligible amount to the electrostatic potential in the repelling region.

CONCLUSION

The concept of using atmospheric gasses as propellant for spacecraft is a conceptually appealing idea due to the mass and volume savings that it can provide, along with increasing the lifetime of low orbit satellites, and extending their capabilities at these altitudes. An analytical model shows that the Molecular Fly-Through Electrostatic Ram Thruster fails to provide the ionization fraction required for successful application. Obtaining the required electron number density while avoiding significant ion losses remains the key issue in employing ambient neutral gas in an electrostatic thruster system.

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